TELESCOPE DESIGNS FOR PRECISION ASTROMETRY

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Abstract

In this paper a design study is described for telescopes to be used in precision astrometry. Specifically, the application is the search for planets outside the solar system. The requirements are stringent because of the need to be spectrally independent, have low distortion, and be free of coma and other asymmetric aberration. The image requirements for astrometry are defined along with two mirror telescope designs for the application. Effects of misalignment and long term stability are considered.

Introduction

The search for planets outside the solar system requires a major advance in astronomical measurements. One of the most promising methods is to look for small periodic motions of stars due to the orbital motion of large planets. For example, the sun moves periodically approximately one stellar radius because of the revolution of jupiter. Such a small motion could be observed for certain neighboring stars by detecting the relative motion against distant background stars, provided relative angular motions of about 50 picoradians could be detected. Methods for obtaining the centroid shift with the required precision are given in Reference 1. For an aperture size of 1 meter, the required precision is about 1 part in 25,000 of the Airy disk. Detection must be made by observations over a period of 5 to 20 years.

One of the major requirements for the image is that it be either circularly or bilaterally symmetric. This means the telescope design should be as circularly symmetric and must reduce coma and higher order asymmetric aberration as much as possible. Symmetric aberration does not shift the centroid directly; but if the illumination is nonuniform, as might happen due to deterioration of the optical coatings, there will be a shift. Distortion does not affect the image energy distribution but causes an uncertainty in the absolute position of the centroid. From the above concerns it will be desired to place the target star at the center of the field for minimum aberration and calibrate out the distortion errors when centroiding the reference stars. Another major requirement is spectral independence. There must not be any shift in the centroid due to changes in wavelength. For this reason the designs will incorporate only reflective elements. The designs will also be limited to two mirrors maximum. This reduces alignment and stability problems as well as overall complexity. It is also desired to operate on a flat medial field. This will minimize the effects of astigmatism and allow the use of flat detector elements.

After an initial investigation showed that the only viable two-mirror flat field designs were extremely long it was decided to look at shorter designs which meant that the medial image surface is no longer flat. It was also decided to double the primary diameter in order to collect more light increasing the signal-to-noise ratio.

Design Approach

The first step in the design approach was to scan the literature for two-mirror aplanatic designs. The aplanat makes a good starting point since it is free from spherical aberration and coma. Most of the recent work on this type of system was done in the late 1960's and early 1970's as part of the space
two-mirror aplanats. The goal was to find a suitable reference wherein equations relating configuration parameters could be used. The reference found most suitable was that by Wetherell and Rimmer. The authors present a general analysis of aplanatic types such as the Ritchey-Chretien as well as a section on flat field aplanats. The flat field aplanats come in three distinct types. These are the gregorian, inverse Cassegrain and Schwarzschild. The Ritchey-Chretien is the aplanatic version of a Cassegrain. One can reject the gregorian and inverse Cassegrain types based on practical considerations. The inverse Cassegrain does not allow access to the image and the secondary is bigger than the primary. The flat field Gregorian has a secondary equal to or greater than the primary and requires a special double sided flat fold mirror to access the image. This leaves the Schwarzschild and Ritchey-Chretien designs as the most viable for our application. The different aplanatic types are shown in Figure 1. The aspects of the Ritchey-Chretien design were investigated in Reference 8.

The main parameter for all the designs is the secondary magnification factor $M$. This is defined as the ratio of the system focal length to the primary focal length. For different values of $M$ one can tabulate many configurations giving an aplanatic design. Since the flat field designs correct for a third aberration there is a unique configuration for each $M$ which gives primary/secondary radii and conic constants, vertex separation, and back focal distance. For the Ritchey-Chretien designs there are many configurations for a given $M$ so it was decided to fix the back image clearance at 1 meter. This sets the focal plane at 1 meter behind the primary. This was chosen as a reasonable value for mounting instrumentation etc. A simple computer program was written to tabulate many configurations based on the equations from Reference 1. A complete computer program for two mirror telescopes was developed by Droessler. For any given $M$ and system focal length one can calculate the mirror radii of curvature, vertex separation, conic constants, obscuration, field curvature as well as 3rd order astigmatism and distortion. The relevant equations are summarized in the Appendix.

The Schwarzschild configurations ranged in length from 13 to 21 meters while the Ritchey-Chretien designs went from 2 to 6 meters. A plot of performance vs. length is shown in Figures 2 and 3 for both types. The scale is the same for both plots. For the Ritchey-Chretien the astigmatism and distortion decrease with length whereas they both increase with length for the Schwarzschild. The Schwarzschild has inherently lower astigmatism and higher distortion than the Ritchey-Chretien. Remember that the previous analysis was based on third order theory. Both types suffer from higher order aberrations. A point was chosen from each performance curve to optimize a configuration and

<table>
<thead>
<tr>
<th>Initial Design Parameters</th>
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<tbody>
<tr>
<td>focal length:</td>
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<tr>
<td>primary diameter:</td>
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<tr>
<td>wavelength:</td>
</tr>
<tr>
<td>field of view:</td>
</tr>
<tr>
<td>aplanatic</td>
</tr>
<tr>
<td>flat field</td>
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<tr>
<td>no length restriction</td>
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<tr>
<th>Subsequent Design Parameters</th>
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<tbody>
<tr>
<td>primary diameter:</td>
</tr>
<tr>
<td>shorter design</td>
</tr>
<tr>
<td>curved image surface</td>
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obscuration. For the Ritchey-Chretien this represents a 4.8 meter long system with an area obscuration of 23. For the Schwarzschild the design is 17.3 meters long with an area obscuration of 15.
These configurations were optimized at F/13 and F/6.5 using the CODE V lens design program from Optical Research Associates. The Ritchey-Chretien was optimized assuming a flat focal plane at F/13 and both flat and curved focal planes at F/6.5. A longer F/13 Ritchey-Chretien was optimized to compare performance vs the shorter designs. Also included is an F/13 parabola for a one mirror baseline comparison. The variables available for optimization are the conic constants, aspheric sag, and image plane curvature and position. Table 2 lists the designs that were optimized.

The final designs depart slightly from perfect aplanatism due to aberration balancing. An aspheric sag was required on both primary and secondary to eliminate residual spherical aberration. Figure 4 shows the final optimized designs to scale.

Table 2. Designs which were selected for optimization.

<table>
<thead>
<tr>
<th>1 meter diameter primary</th>
<th>2 meter diameter primary</th>
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<tbody>
<tr>
<td>F/13 Parabola</td>
<td>F/6.5 Schwarzschild</td>
</tr>
<tr>
<td>F/13 Schwarzschild</td>
<td>F/13 Ritchey-Chretien</td>
</tr>
<tr>
<td>F/13 Ritchey-Chretien</td>
<td>F/6.5 Ritchey-Chretien</td>
</tr>
<tr>
<td></td>
<td>F/6.5 Ritchey-Chretien (curved field)</td>
</tr>
</tbody>
</table>
Results

Table 3 shows the results of the 1 meter diameter primary designs. The F/13 parabola is included for reference. The collection area is roughly equal for both the Ritchey-Chretien and Schwarzschild designs. The parabola suffers mainly from coma which cannot be corrected. Since the Schwarzschild operates at the medial field it minimizes the effects of astigmatism and therefore has a highly symmetric spot. Only a small amount of residual coma is present. The astigmatism in the Ritchey-Chretien causes the image to be elongated but it will remain acceptably symmetric. The geometrical spot diagrams are given in Figure 5 for all three one meter primary designs. The error in the centroid due to distortion is nine times greater in the Schwarzschild. So there is a direct tradeoff in performance between distortion and astigmatism between the two types.

In the 2 meter diameter primary designs the f-number is 6.5 and the diffraction spot diameter is 7.9 microns. The relative performance is shown in Table 4. The Schwarzschild configuration maintains a highly symmetrical spot even though the aberrations increase over the F/13 design. Figure 6 shows that the Ritchey-Chretien exhibits unacceptable astigmatism and residual coma on a flat image surface. Therefore, operation on a curved image surface was investigated. Figure 7 shows the comparison between the F/6.5 flat field Schwarzschild and curved field Ritchey-Chretien designs. One of the main comparison points here involves the image symmetry. One can see from the plots that although the F/6.5 Schwarzschild has less aberration and therefore a smaller geometrical image it suffers from residual 5th order coma. The F/6.5 curved field Ritchey-Chretien has very good symmetry in the spots with virtually no asymmetric aberration present. It should be noted that going to a longer Ritchey-Chretien at F/13 with a 2 meter primary resulted in worse performance than either a F/13 one meter primary design or a F/6.5 two meter primary design.
Alignment Sensitivity

Alignment sensitivity will be discussed here in primarily a qualitative fashion since no specific tolerances regarding image quality were given. The primary is assumed to constitute the reference axis of the system. Therefore, the concern here is the effect of secondary tilt and decenter. It will also be assumed that the usual secondary mounting configuration applies. This is where the secondary is mounted such that there is no coma introduced for any given tilt and decenter. Remember that these systems are aplanatic so there is no spherical aberration or coma present before perturbation and none is generated after. The medial field will, in general, experience a tilt and axial displacement. The amount of each depends on the astigmatism present. This means the Ritchey-Chretien is more sensitive than the Schwarzschild in this regard. The astigmatism field will become binodal. This means that there will be two places in the field where the astigmatism vanishes. In general, neither of them will occur at the center of the field. Therefore the target star at the center of the field will see astigmatism which will elongate the spot but not introduce asymmetry.

Perhaps the most interesting effect occurs with distortion. For an arbitrary tilt or decenter the distortion will generate a scale change, lateral field shift, anamorphism, and asymmetric tangential and radial distortion. These effects are proportional to the amount of distortion present in the unperturbed system. This means the Ritchey-Chretien is much less sensitive than the Schwarzschild to distortion effects. The anamorphism and radial and tangential effects are with respect to the entire field. The local effect is to shift the centroid. This shift is a function of the field location and is not radially or bilaterally symmetric with respect to the field center. A thorough treatment of the effects of secondary tilt and decenter would be required for a more advanced study. The relative effects of secondary motion about its vertex is given in Table 5.

The long term stability is difficult to quantify. Certainly a major concern will be the integrity of the optical coatings. Also of concern is any degradation of the mirror surface due to micrometeors and other debris. Pointing is usually done with a small fine guidance telescope and the optical axis misalignment over time is a concern. Precise positioning of the telescope is made easier by a mechanical structure that has a low moment of inertia. This may favor a short design in the long run since less energy must be spent to keep it aligned.
Figure 5. Spot Diagrams for optimized 1 meter primary designs.
Conclusions

The preliminary results of the design study show there are two basic telescope designs that can be implemented for the astrometric task at hand. These are the flat field Schwarzschild and the Ritchey-Chretien. The tradeoffs besides length are flat vs. curved detector and primary diameter. The first observation is that the limiting aberration in the unperturbed system is astigmatism. This is a result in agreement with Reference 9. In order to reduce the effect it is necessary to operate at the medial field. The other major concern is distortion. While a fixed amount can be calibrated out the effect of tilt and decenter may be troublesome.

Based on the results some preliminary recommendations can be made. The first is that if length is not a concern then the Schwarzschild design will give the best performance on a flat field. If length or distortion error is a prime concern then the Ritchey-Chretien is the choice. The above applies to...
either a 1 meter or 2 meter primary design. If one chooses the 2 meter primary Ritchey-Chretien then 
the detector must be curved to give acceptable performance. One thing to consider is that the secondary 
for the Schwarzschild is concave making it easier to test.

There are several advantages to be gained by going to a 2 meter diameter primary mirror. The 
first is that the focal plane irradiance increases by a factor of 16. This becomes a factor of 64 if the 
focal length is kept constant since the f-number and the image diameter are halved. Therefore the 
signal-to-noise ratio will be substantially increased. The smaller spot means a shorter grating period for 
the centroid detection scheme and therefore a higher sampling rate increasing the SNR10. The obvious 
disadvantage is the need to fabricate and test larger mirrors and the increase in telescope diameter and 
weight.

![Field focus and ray fan plots for Schwarzschild and Ritchey-Chretien designs.](image)
Figure 8. 2 Meter Diameter Primary Designs.

Table 5. Secondary Sensitivity to Vertex Misalignment

<table>
<thead>
<tr>
<th>Tilt (1 mrad)</th>
<th>Decenter (1 mm)</th>
<th>Despace (1 mm)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>R</td>
<td>S</td>
</tr>
<tr>
<td>δW_{131}</td>
<td>-.24</td>
<td>-.15</td>
</tr>
<tr>
<td>δW_{222}</td>
<td>446</td>
<td>331</td>
</tr>
<tr>
<td>δW_{20}</td>
<td>446</td>
<td>331</td>
</tr>
</tbody>
</table>

R - Ritchey-Chretien
S - Schwarzschild

Displacements are positive
Values are in microns

δW_{131} is the change in coma
δW_{222} is the change in astigmatism
δW_{20} is the change in focus

Further work that could be done is to quantify fully the effects of secondary tilt and decenter. This is especially useful in the case of distortion. If absolute distortion correction is required one can investigate aplanatic designs that correct for distortion instead of field curvature. Solutions exist using the Gregorian configuration that allow this.
Appendix

This section lists the relevant equations that were used to calculate the telescope configurations based on third order theory. The equations are obtained from Reference 1.

**Preliminary Definitions**

- $f$ is the system focal length
- $f_p$ is the primary focal length
- $F$ is the system $f$-number
- $F_p$ is the primary $f$-number
- $D_p$ is the primary diameter
- $B$ is the primary vertex back focal clearance

**General Astigmatic Equations**

System $f$-number:

$$ F = \frac{f}{D_p} \quad (1) $$

Primary $f$-number:

$$ F_p = \frac{f_p}{D_p} \quad (2) $$

Secondary magnification:

$$ M = \frac{F}{F_p} \quad (3) $$

Back focus parameters:

1. $\eta = \frac{B}{D_p}$ and
2. $\beta = \frac{B}{f_p}$

$$ (4) \quad (5) $$

Vertex separation:

$$ S = \frac{D_p(F-\eta)}{M+1} \quad (6) $$

Primary radius:

$$ R_p = -2D_pF_p \quad (7) $$

Secondary radius:

$$ R_s = \frac{2MD_p(F_p+\eta)}{1-M^2} \quad (8) $$

Secondary diameter:

$$ D_s = D_p \left( \frac{F_p + \eta - \frac{uF_p}{F_p+\eta}}{F_p+F_s} \right) \quad (9) $$

Area obscuration:

$$ \epsilon = \left( \frac{D_s}{D_p} \right)^2 \quad (10) $$

Primary conic constant:

$$ K_p = \frac{-2(F_p+\eta)}{M^2(F_p-\eta)} \quad (11) $$

Secondary conic constant:

$$ K_s = \frac{2M(M+1)}{(M^2-1)(F_p-\eta)} + \left( \frac{4M}{(M-1)^2} \right) \quad (12) $$

Medial surface radius:

$$ R_m = \frac{-M^2f_p(1+\beta)}{M(M+1)-\beta}\beta \quad (13) $$

**Flat Field Equations**

$$ R_m = \infty \text{ therefore;} \quad (14) $$

$$ \beta = \frac{M^2}{M-1} $$
References

1)  C. Huang and G. Lawrence, "Focal Plane Techniques for Picoradian Measurements of Stellar Images," SPIE Vol 818, August 1987


8)  E. Mendoza and A. Buffington, "Investigation of the Properties of the Ritchey-Chretien," Center for Astrophysics and Space Sciences, Univ. of California at San Diego, La Jolla, California (1986).
